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Metacognitive aspects of solving function problems

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Abstract

The purpose of this study is to analyze the role of meta-cognition while solving function problems and to develop suggestions for classroom practice. Especially, the first of three objectives to be attained is to monitor meta-cognitive behaviors of prospective teachers, who study at the department of “teaching mathematics at primary schools”, while solving problems; the second is to evaluate the importance of meta-cognition in solving problems; and the third is to analyze typical errors, challenges, successes and strategies chosen for solving function problems. The analysis of the questionnaires filled out by students, as well as students’ solutions related to the problems, provide an understanding for the meta-cognitive aspects of solving function problems.

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1. Introduction

Problem is a case which contains open-ended questions, which attracts the attention of an individual and which the individual does not have the necessary algorithm and method knowledge to solve these questions (Bloom & Niss, 1991). According to John Dewey, a problem is something that confuses human mind, challenges to it and obscures the belief. If a case is to be a problem, it should be encountered for the first time and it should be new. Problems are things that obscure belief and confuse minds (Baykul, 2004). Problem solving is to know what should be done when it is not known what to do. Problem solving process, which is only perceived as reaching the correct solution, in fact, covers wider mental process and skills than this. Problem solving is finding a way and getting rid of the difficulty in addition to reaching the solution (Polya, 1957).

Various factors may influence an individual while solving a problem. Factors such as whether the problem is suitable for the age of the person involved, whether he/she has prior knowledge and skills to find the solution, and personal characteristics can be effective in solving problems. Similarly, it is seen that meta-cognitive levels can also be effective in solving problems (Gelbal, 1991).

Metacognition is being aware of one’s own cognitive structure and learning characteristics (Senemoğlu, 2011). According to Flavell (1979), metacognition is a system which organizes information, experiences, objectives and strategies. Metacognition, which means thinking about thinking, generally covers various skills that are inter-related to thinking and learning, which are critical thinking, reflective thinking, problem-solving and making a decision. Individuals, who have more developed metacognitive skills, are also better problem solvers, decision makers and critical thinkers than others (Dawson, 2008).

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In its broadest sense, metacognition is human's awareness of mental operations like perception, remembering and thinking and ability to control them (Hacker & Dunlosky, 2003). Schoenfeld (1987), in his study about metacognition, divided one's mental behaviours that are important for problem solving into three categories: *information about one's thinking process, controlling one's actions or self-regulation, beliefs and attitudes*.

Self-regulation, which is important for problem solving and covers metacognitive strategies, is students' ability to direct the effort for completing an academic task in the classroom. In addition to that, self-regulation is a combination of cognitive strategies like repeating, explanation and organizing that are used for learning, remembering and understanding (Pintrich & De Groot, 1990). In this study, the aim is to analyze the role of metacognitive skills and to develop suggestions for classroom practice while solving function problems. The reason for choosing function problems is the necessity for students to use metacognitive skills while solving function problems. In this sense, it is thought that this study is important in terms of focusing on both problem solving process and metacognitive skills in this process.

2. Method

2.1. Design of the Research

This study is a qualitative case study, which was conducted to find results related to a specific situation. A case study is an empirical research method; (1) which deals with an up-to-date case in its own content, (2) in which there is not a clear cut borders between the case and the content of the case and (3) which is used when there is more than one evidence and data source (Yin, 1984).

2.2. Study Group

The study was conducted with 39 (78%) female and 11 (22%) male students, who study at the Education Faculty of Kocaeli University. All participating students were freshman students at the department of "teaching mathematics at primary schools".

2.3. Data Collection Tools, Collecting Data and Data Analysis

All the participants were asked to solve two function problems which were prepared by the researchers. In the first problem, the solution could only be found by using compound function feature of an algebraic operation. In the second one, initially the data relating to the given function needed to be analyzed and then first the analysis required to be modeled by using graphics and later on the features of absolute value in conceptual dimension required to be converted into a graphical model. The second problem required a little more thinking process than the first one, since it required more knowledge and had the dimension of information transfer.

In line with the main purpose of the study, immediately after students had solved two problems, '*Using Metacognitive Skills Questionnaire*', which was adapted from Biryukov (2002) and consisted of 14 questions defining cognitive and meta-cognitive behaviours, was conducted. Students were given 50 minutes in total: 30 minutes for solving the problems and 20 minutes for answering the questionnaire. The data obtained as a result of the implementation was analyzed descriptively. At the first step of the analysis, participating students' responses for the problems were analyzed. The items of the questionnaire that had been filled out by students were analyzed one by one within the framework of these solution protocols.

3. Findings and Comments

At the first stage of the study, two function problems were given to students. The following findings were observed after the students' responses had been analyzed within this context; 22 (44%) *students solved both problems*, 9 (18%) *students solved only the first problem*, 10 (20%) *students solved only the second problem*, 9 (18%) *students could not solve any of the problems*. Immediately after the students had solved the problems, they

filled out a questionnaire containing 14 questions, which defined their cognitive and meta-cognitive behaviours while solving problems. The results of the analysis of the answers of the questionnaire and problems are provided below:

1st Item of the Questionnaire	Yes	No	Not Sure
I read the problem more than once	22	25	3

44% of the participants answered this item as 'yes'. 7 participants from among those who answered this item as 'yes' also solved both problems, while 3 of them could not solve any of the problems, 6 of them solved the first problem and 6 of them solved the second problem. After solution protocols, it was seen that 13 participants out of those who answered this item as 'no' solved both problems, 3 of them solved only the first problem, 4 of them solved only the second problem and 5 of them could not solve any of the problems.

2nd Item of Questionnaire	Yes	No	Not Sure
I checked what the problem was asking to me	43	5	2

The expression given in the second item shows meta-cognitive self-regulation behaviour while solving a problem and verifies whether the problem is understood properly or not. Most of the participants (86%) answered this item as 'yes'. While 2 participants out of those who answered this item as 'no' solved both problems, 2 of them solved only the first problem, and one of them solved the second problem successfully. While one student from among two who answered this item as 'not sure' solved only the first problem, the other one could not solve any of the problems.

3rd Item of the Questionnaire	Yes	No	Not Sure
I evaluated how much time I need for solving this kind of problem	12	35	3

70% of the participants answered this item as 'no'. It was seen that most of them did not consider any time limit for solving the problems. As a matter of fact, it is interesting to see that students who are constantly in a system that is surrounded by examinations did not feel any necessity to check the time.

4th Item of the Questionnaire	Yes	No	Not Sure
I showed the problem schematically	34	12	4

Creating models for solving a problem takes part in self-regulation behaviours in mathematics. Creating models by individuals with their own experiences is an example for meta-cognitive behaviour and is helpful for understanding the relationship between the conditions and the problem to be solved. In this regard, as a result of the analysis, while 12 participants said 'no', 4 participants said 'not sure'.

5th Item of the Questionnaire	Yes	No	Not Sure
I tried to recall whether I solved a similar problem before or not	36	14	0

14 of the participants answered this item as 'no'. While 6 of them solved both problems correctly, 4 of them solved only the first problem, and 1 of them solved only the second problem successfully. 3 of them could not solve any of the problems. 16 participants from among those who answered this item as 'yes' solved both problems successfully. While 6 of them could not solve any of the problems, 5 of them solved only the first problem and 9 of them solved only the second problem successfully.

6th Item of the Questionnaire	Yes	No	Not Sure
I developed a strategy for solving the problem	28	16	6

While self-regulation behaviours enable understanding the concepts in mathematics, they also consist of using knowledge for achieving targets. Metacognitive analysis and control provide developing strategy for achieving these targets. In this regard, 3 participants out of 16 who answered this item as 'no' solved both problems. While 4 of them solved only the first problem and 5 of them solved only the second problem, 4 of them could not solve any of the problems.

7th Item of the Questionnaire	Yes	No	Not Sure
I did not know where to start	6	38	6

One of the participants out of those who answered this item as ‘yes’ solved both problems successfully, and while 2 of them solved only the first problem successfully, 1 of them solved only the second problem. The other 2 could not solve any of the problems. 18 participants out of those who answered this item as ‘no’ solved both problems successfully. While 7 of them solved only the first problem successfully, 7 of them solved only the second problem successfully. 5 of the participants could not solve any of the problems.

8 th Item of the Questionnaire	Yes	No	Not Sure
I faced some difficulties while solving problem (If your answer is ‘yes’, define the features of the difficulty)	36	14	0

Among 21 participants who declared that he or she faced some difficulties, all except 2 of them defined the difficulty. For instance; ‘*There were some things that I could not remember*’, ‘*I could not remember whether the signs had changed or not*’. The participants who said ‘yes’ generally expressed the concept of ‘remembrance’. Mentioning the concept of remembrance, which is one of strategies (like questionnaire, interview, stimulated recall and recall) used for testing the meta-cognitive skills suggests the presence of the meta-cognition.

9 th Item of the Questionnaire	Yes	No	Not Sure
I found one of my mistakes while solving the problem and corrected it (If your answer is yes, define the mistake).	14	34	2

It was seen that 7 of the participants out of those who answered this item as ‘yes’ did not provide any explanation but 7 of them defined their mistakes. For instance; ‘*In the second question, the arms would be parabolic*’, ‘*I realized I was following the wrong way*’. It was seen that nine of the participants who found their mistakes solved both problems correctly, two of them solved only the first problem and three of them solved only the second problem.

10 th Item of the Questionnaire	Yes	No	Not Sure
I thought how the solution was going on	42	6	2

It is important that 84% of the participants answered this item as ‘yes’, which shows the cognitive evaluation towards a solution. 19 of the participants out of those who answered ‘yes’ solved both problems. While 7 of them could not solve any of the problems, 7 of them solved only the first problem, and 9 of them solved only the second problem.

11 th Item of the Questionnaire	Yes	No	Not Sure
I tried different approaches to solve the problem	21	27	2

Six participants out of those who answered this item as ‘yes’ solved both problems successfully. While 14 participants out of those who answered this item as ‘no’ solved both problems successfully, 4 of them could not solve any of the problems.

12 th Item of the Questionnaire	Yes	No	Not Sure
I asked myself whether my answers were meaningful or not.	33	12	5

While solving mathematical problems, it is important to check whether the answers are meaningful or not and this is an example for a meta-cognitive behaviour. In this regard, 66% of the participants answered this item as ‘yes’. 14 participants out of those who answered this item as ‘yes’ solved both problems successfully. While 6 of them solved the first problem successfully, 9 of them solved the second problem successfully. 4 of them could not solve any of the problems.

13 th Item of the Questionnaire	Yes	No	Not Sure
I checked my calculations to be sure that they were correct.	27	16	7

Thirteen participants out of those who answered this item as ‘yes’ solved both problems successfully. Four of them solved only the first problem and 7 of them solved only the second problem successfully. 3 of them could not solve any of the problems.

14 th Item of the Questionnaire	Yes	No	Not Sure
I thought whether there was something that I should especially pay attention or not in the information given in the problem (If ‘yes’, define it).	21	22	7

While 3 participants out of those who answered this item as ‘yes’ did not provide any explanations, others provided explanations. For instance; ‘*I paid attention to absolute value sign*’, ‘*I checked whether the graphic was absolute value graphic or not*’. It was seen that the expressions like ‘paid attention’, ‘checked’ and ‘thought’ were frequently used. In this regard, it can be said that the participants used meta-cognitive skills.

4. Conclusion, Discussion and Implications

Increase in metacognitive skills, which is a key factor in creating and maintaining successful learning, also increases the learning improvement. It is stated that the achievement would improve by having students who are aware of their own thoughts in the problem solving process (Senemoğlu, 2011). As a result of the findings of this study, comparing the answers of students related with the solution of function problems shows the importance of the emergence of the cognitive experiences. Individuals can be more successful in problem solving by having metacognitive experiences. In this study, it was seen that 22 participants who solved both questions successfully also said ‘yes’ to the 1st, 2nd, 4th, 6th, 10th, 12th, 13th and 14th items of the questionnaire. It is seen that they used metacognitive skills that made them successful in problem solving. In a number of studies, it was seen that there is a significant relationship between problem solving and metacognitive skills; these skills improve the success in problem solving and so that students are able to organize their mental process more effectively (Flavell, 1976; Schoenfeld, 1985; Garofalo & Lester, 1985; Özsoy, 2008).

There are two important products of problem solving. The first one is the development of special strategies and rules specific to subject which is taught and the other one is the improvement of mental processes and approaches that can be used for creating a rule or formula. Students learn to solve new kind of problems by organizing former strategies and to create new strategies in a problem situation (Olkun & Toluk, 2004: 44). Also in this study, it is seen that the expressions in Question 4 and Question 6, which are for schematic representation and strategy development respectively, are important for the discovery of metacognitive skills in successful problem solving. It appears that 11 participants out of 20 who answered both items as ‘yes’ solved both problems successfully.

The findings of this study revealed the importance of metacognition in solving mathematical problems. In this regard, it is important that teachers have their students perform activities that reveal their metacognitive skills in problem solving process.

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